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Wave-equation Common Image Gathers and their Combination with Nonlinear Ray-based Tomography

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SUMMARY

It has been reported that if the migration velocity is erroneous, then different migrations lead to different RMO in common image gathers. However, it is widely assumed that the travel-time along the 'migration ray' is the same as the actual travel-time for the corresponding source and receiver positions. Travel-times are kinematic invariants for Kirchhoff migration, but not generally for WE migration. In the current paper we show that (i) RMO from dual domain WE CS migration is the same as that for double-square-root migration, (ii) for both migrations, RMO of offset slowness gathers satisfies the condition of preserving the plane-wave travel-times (PWTT), (iii) for CS migration, offset-slowness gathers obtained by plane-wave decomposition from source and receiver sides are the same as those calculated by the Radon transform of extended images, and finally, (iv) PWTT are used when combining RMO from CS WE migration with non-linear ray-based tomography. The described tomography does not involve any linearization and does not require closeness of the initial model to the true one. We illustrate the approach with a synthetic 3D example, where we obtain accurate inversion results after a single iteration of migration, RMO analysis, reconstruction of PWTT, and tomography.

Introduction

Velocity model building for advanced depth imaging in complex areas is still one of the main challenges in seismic exploration. One of the most practical directions for velocity inversion is based on using residual move-out (RMO) in common image gathers from Kirchhoff or wave-equation (WE) migrations in a combination with ray-based or WE tomography (Jones et al, 2008; Nguyen et al, 2008). Such a combination requires a clear understanding of the properties of RMO. It has been reported (Montel and Lambaré, 2013) that if the migration velocity is erroneous, then different migrations lead to different RMO. However, it is still widely assumed that the travel-time along the 'migration ray' is the same as the actual travel-time for the corresponding source and receiver positions. Travel-times are kinematic invariants for Kirchhoff migration, but not generally for WE migration. In the current paper we show that (i) RMO from the dual domain WE CS migration is the same as that for the double-square-root (DSR) migration, (ii) for both migrations, RMO of offset slowness gathers satisfies the condition of preserving plane-wave travel-times (PWTT), (iii) for CS migration, offset-slowness gathers obtained by plane-wave decomposition from source and receiver sides are the same as those calculated by Radon transform of extended images, and finally, (iv) PWTT are used when combining RMO from CS WE migration with non-linear ray-based tomography. We illustrate the approach with a synthetic 3D example, where we obtain accurate inversion results after a single iteration of migration, RMO analysis, reconstruction of PWTT, and tomography.

Properties of RMO from WE migration

Probably the easiest way to introduce the concept of 'PWTT for tomography' is to consider the impulse response for DSR migration. Assume that the data (2D for simplicity) is a spike at common mid-point (CMP) x_0, offset h_0, and time t_0. Then in the frequency domain:

$$D(x,h,\omega)=\delta(x-x_0)\delta(h-h_0)\exp\{i\omega t_0\}, \quad (1)$$

The Radon transform of (1) over offset h and CMP x is:

$$\tilde{D}(p_d,p,\omega)=\exp\{i\omega(t_0-ph_0-p_d x_0)\}, \quad (2)$$

where p_d and p are CMP-slowness and offset-slowness respectively. Then the result of DSR migration in a homogeneous medium with the velocity v_m is:

$$\text{Image}(x_m,z_m,p)=\int_\omega \frac{\omega}{2\pi}\int_{p_d}\tilde{D}(p_d,p,\omega)\exp\{i\omega[-(q_{sm}+q_{rm})z_m+p_d x_m]\}dp_d d\omega=$$

$$=\int_\omega \frac{\omega}{2\pi}\int_{p_d}\exp\{i\omega[t_o-ph_0-(q_{sm}+q_{rm})z_m+p_d(x_m-x_0)]\}dp_d d\omega, \quad (3)$$

where q_{sm} and q_{rm} are vertical slownesses from the source side and the receiver side respectively:

$$q_{sm}=\sqrt{1/v_m^2-p_s^2},\; q_{rm}=\sqrt{1/v_m^2-p_r^2},\; p_r=p+p_d/2,\; p_s=p-p_d/2. \quad (4)$$

From (3) we obtain the following two conditions to form the image:

Balance of plane-wave travel-times (for the stationary value of p_d, see the next condition):

$$\tau_0=t_o-ph_0-p_d x_0=(q_{sm}+q_{rm})z_m-p_d x_m=t_m-ph_m-p_d x_{ms}=\tau_m, \quad (5)$$

where t_m is the travel-time along the 'migration ray' corresponding to p_r and p_s; x_{ms} and h_m are CMP coordinate and offset at the surface, also corresponding to the 'migration ray', see Figure 1. τ_0 and τ_m define PWTT from the input data and from the result of migration.

Stationarity condition: The image (3) is formed if

$$\frac{\partial\varphi}{\partial p_d}=0, \text{ where } \varphi=t_o-ph_0-(q_{sm}+q_{rm})z_m+p_d(x_m-x_0). \text{ Therefore, } z_m\frac{\partial(q_{sm}+q_{rm})}{\partial p_d}-(x_m-x_0)=0.$$

Taking into account (4), we obtain: $\frac{z_m}{2}(\text{tg}\,\alpha_r-\text{tg}\,\alpha_s)+x_m=x_0$, or $x_{ms}=x_0$, (6)

where x_{ms} is the surface CMP position corresponding to the 'migration ray'; α_s and α_r are the angles between the source- and the receiver-side rays respectively and the vertical axis (see Figure 1). Conditions (5) and (6) are the basis for combining ray-based tomography and RMO of offset-slowness gathers from the WE migration. They have a similar form for an arbitrary 2D structure and for the CS WE migration. The extension for the 3D case is straightforward.

One can check that the image from the CS migration for the given offset slowness p and for the same spike in the data is similar to (3). The only difference is in the additional amplitude term appearing in the CS migration result due to the weights of point-source plane-wave decomposition. This additional term does not affect the results (5) and (6).

Consider now CS migration in the homogeneous media with the velocity v_m of the data from a single plane dipping reflector. The true velocity is v and the true depth of the reflector at x=0 is H. In the following formulas we drop amplitude terms, since they are not important for the current analysis. All formulas are in the frequency domain. The CS data after the Radon transform over offset are:

$$D(x_s, p_r) = \exp\left\{i\omega\left[H(q_r + q_s) - p_s x_s\right]\right\}, \tag{7}$$

where q_r and q_s are vertical slownesses corresponding to the true velocity; p_s is the horizontal source-side slowness, uniquely defined by p_r, the true velocity v, and the reflector dip. The receiver wave-field back-propagated to the image point (x_m, z_m) is:

$$U(x_s, p_r, x_m, z_m) = \exp\left\{i\omega\left[H(q_r + q_s) - p_s x_s - z_m q_{rm} + p_r x_m\right]\right\}. \tag{8}$$

The source wave-field above the image point and for the given horizontal slowness p_{sm} is:

$$S(x_s, p_{sm}, x_m, z_m) = \exp\left\{i\omega\left[z_m q_{sm} + (x_m - x_s) p_{sm}\right]\right\}. \tag{9}$$

Then the image from the source at x_s is defined as:

$$\mathrm{I}(x_s, p_{sm}, p_r, x_m, z_m) = US^* = \exp\left\{i\omega\left[H(q_r + q_s) - z_m(q_{rm} + q_{sm}) + x_m(p_r - p_{sm}) + x_s(p_{sm} - p_s)\right]\right\},$$

where S^* denotes the complex conjugate of S. The total image is obtained by integrating over all sources. Taking into account that $\int \exp\left\{i\omega(p_{sm} - p_s)x_s\right\}dx_s = \delta(p_{sm} - p_s)$, we obtain:

$$\tilde{\mathrm{I}}(p_r, x_m, z_m) = \exp\left\{i\omega\left[H(q_r + q_s) - z_m(q_{rm} + q_{sm}) + x_m p_d\right]\right\}, \tag{10}$$

where p_s (required to calculate q_s, q_{sm} and p_d, see (4)) is uniquely defined by p_r, the true velocity, and the true reflector dip. Since $p_d/(q_r + q_s) = \mathrm{tg}\alpha$, and since p_r and p_s define the offset slowness p, we finally obtain:

$$\tilde{\tilde{\mathrm{I}}}(p, x_m, z_m) = \exp\left\{i\omega\left[H_x(q_r + q_s) - z_m(q_{rm} + q_{sm})\right]\right\}, \tag{11}$$

where H_x is the true reflector depth at x_m. This is the same equation that one immediately obtains by using DSR migration (where p_r and p_s are found from p, the velocity, and the dip). For the simple case of a plane dipping reflector, PWTT are transformed into vertical travel-times, which are required to be preserved to form the image. Here we have demonstrated that RMO of offset-slowness gathers from CS migration is the same as for DSR migration. The equivalence of source-receiver migration and of CS migration for extended images has been proven by Biondi (2003).

Consider now an extended image $M(h, x_m, z_m)$ for the same data from the dipping reflector. Once again, all equations below are in the frequency domain, and all amplitude factors are neglected. From (8) and (9) it follows that:

$$M(h, x_m, z_m) = \iiint U(x_m + h/2, z_m) S^*(x_m - h/2, z_m)\, dp_r dp_{sm} dx_s =$$
$$= \iint \exp\left\{i\omega\left[H(q_r + q_s) - z_m(q_{rm} + q_{sm}) + x_m(p_r - p_{sm}) + h(p_r + p_{sm})/2\right]\right\}\cdot$$
$$\cdot\left[\int \exp\left\{i\omega x_s(p_{sm} - p_s)\right\}dx_s\right]dp_r dp_{sm}. \tag{12}$$

The last integral in (12) over x_s leads to the delta-function $\delta(p_{sm} - p_s)$. Therefore, we finally obtain:

$$M(h, x_m, z_m) = \int \exp\left\{i\omega\left[H_x(q_r + q_s) - z_m(q_{rm} + q_{sm})\right]\right\}\cdot \exp\left\{i\omega ph\right\}dp, \tag{13}$$

(see derivation of (11)). Comparing (13) and (11), we conclude that offset slowness gathers from the CS migration can be obtained by the Radon transform of extended images. The same is valid for a general 2D structure. The proof is lengthy and will not be given here. Note that the Radon transform is applied in the frequency domain within the migration (before final summation over frequencies). Therefore, the procedure is probably not as efficient as post-migration slant-stacking (Rickett and Sava, 2002). Accuracy versus efficiency should be further investigated. Figure 2 illustrates that RMO of p-gathers from the CS migration is the same as theoretical RMO from DSR migration. The p-gathers have been calculated by the Radon transform of extended images within the migration.

Ray-based tomography combined with RMO from WE migration

As shown in the previous section, PWTT (equation (5)) is a kinematic invariant, which is used for velocity inversion. In our procedure we assume that the model is layer- or block-based, where within each layer velocity varies laterally, but not vertically. For simplicity, we outline our inversion scheme for the 2D case. The 3D extension is straightforward. **1)** From the results of CS migration with p-gathers, we pick the main horizons used in inversion and corresponding RMO. Since the RMO dependency on p is an even function, we assume that this function can be approximated by a hyperbola, where at each point along the horizon the hyperbolic coefficient is estimated based on maximization of semblance. Figure 4 illustrates the concept for the synthetic data from the 3D model in Figure 3. **2)** The results of picking are used to solve the direct problem of reconstructing PWTT at the surface. For each p and for each point along the horizon (with p-dependent depth) we shoot source and receiver rays to the surface. The directions of the rays are defined by p, the apparent (p-dependent) local dip, and the local velocity used in the migration. Based on the results of ray tracing, we reconstruct PWTT $\tau_m(x, p_m)$ for corresponding CMP positions at the surface (see (5) and (6)); p_m is the offset slowness at the surface, which is generally different from the p at the image point. **3)** Reconstructed PWTT are used for layer-stripping inversion. For the current layer and for each trial velocity within the pre-defined range, we shoot back the normal rays (p=0) until the balance of zero-offset travel-times is reached. The end points of normal rays define the (velocity-dependent) bottom of the current layer. Finally, for each x at the surface (position of the normal ray at the surface) and for each p_m, we find a corresponding offset (reflection point along the bottom of the current layer) and calculate the corresponding 'synthetic' PWTTs. The optimal velocity (in the vicinity of the end point of the normal ray) is the one that provides the minimum misfit between reconstructed and 'synthetic' PWTTs. Figure 5 shows the result of inversion for the second layer of the model, and Figure 6 compares the final inversion results with the true model. Although not ideal, the inversion results are quite accurate. The discrepancies are probably related to non-optimal parameterization of the residual move-out. In the given synthetic example a single iteration of migration with the constant velocity has been used for picking, reconstruction, and inversion. For data from complex velocity models, this sequence can be used in iterations, so that 'true' velocities are assumed above a given datum. In this case we can avoid ray-tracing through the complex overburden, since wave-field redatuming is a built-in step in WE migration.

Conclusions

Plane-wave travel-times are kinematic invariants for DSR and CS WE migrations in the dual domain. These travel-times are used when combining RMO on offset-slowness gathers with ray-based tomography. The described tomography does not involve any linearization and does not require closeness of the initial model to the true one. In the given synthetic example, accurate results are obtained by a single iteration of migration, RMO analysis, reconstruction of PWTT and inversion.

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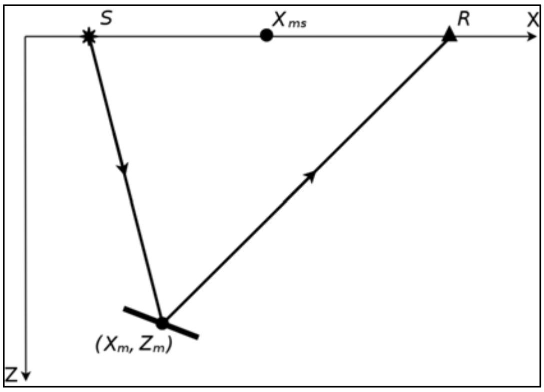

Figure 1 *Schematic of rays*

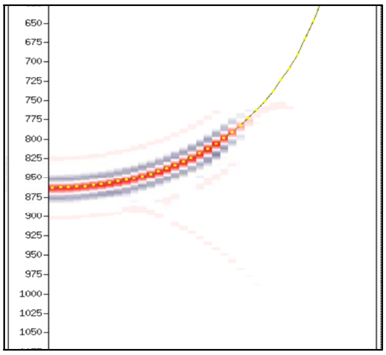

Figure 2 *Slowness-domain common-image gather from CS WE migration with theoretical RMO from DSR migration superimposed.*

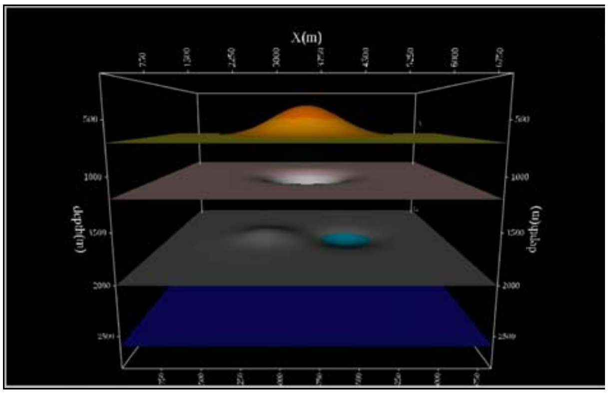

Figure 3 *True model. Velocities in layers (from top to bottom): 2000,3000,2600,3500 m/sec.*

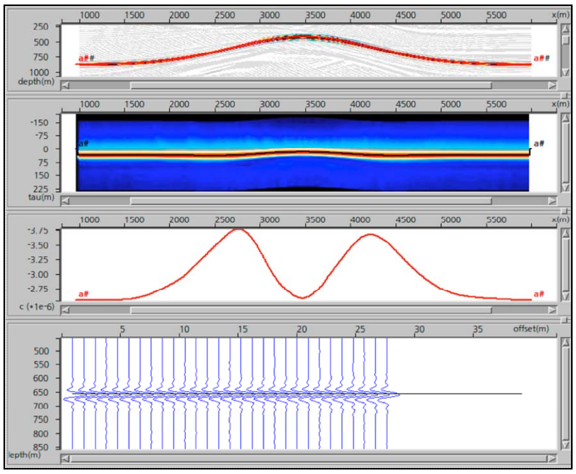

Figure 4 *Central inline, RMO analysis for the 1st interface (from top to bottom): Image, RMO semblance, RMO parameter, and RMO-corrected P-gather.*

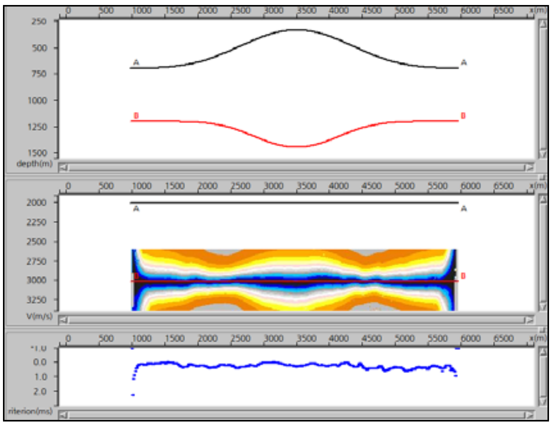

Figure 5 *Central inline, inversion results for the 2nd interface (from top to bottom): depth, velocity (with an objective function in a color-scale), and PWTT misfit.*

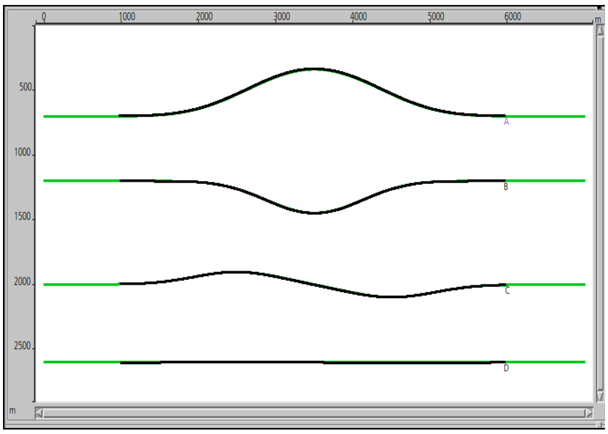

Figure 6 *Results of inversion (black) and true (green) depth models. A single migration with the constant velocity (2500 m/sec) has been used for picking, reconstruction of plane-wave travel-times, and inversion.*